1. The parametric equations of a curve are

$$x = 2t + \sin 2t, \quad y = \ln(1 - \cos 2t).$$

Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \operatorname{cosec} 2t$. [5]

B1 $\dfrac{dy}{dt} = \dfrac{2\sin 2t}{1-\cos 2t} = \dfrac{4\sin t\cos t}{2\sin^2 t} = \dfrac{2\cos t}{\sin t}$

B1 $\dfrac{dx}{dt} = 2 + 2\cos 2t = 4\cos^2 t$

M1 $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{\frac{2c}{s}}{4c^2} = \dfrac{1}{2cs}$

$= \dfrac{1}{2\sin t\cos t}$

M1 $= \dfrac{1}{\sin 2t}$

A1 $= \operatorname{cosec} 2t$

2. A curve has equation $y = \dfrac{e^{3x}}{\tan \frac{1}{2}x}$. Find the $x-$ coordinates of the stationary points of the curve in the interval $0 < x < \pi$. Give your answers correct to 3 decimal places. [6]

$$\frac{dy}{dx} = \frac{3e^{3x} \cdot \tan\frac{1}{2}x - e^{3x} \cdot \frac{1}{2}\sec^2\frac{1}{2}x}{\left(\tan\frac{1}{2}x\right)^2}$$

M1 A1

$3\tan\frac{1}{2}x - \frac{1}{2}\left(1 + \tan^2\frac{1}{2}x\right) = 0$

M1 $6t - 1 - t^2 = 0$

$t^2 - 6t + 1 = 0$

M1 $t = \dfrac{6 \pm \sqrt{32}}{2} = 3 \pm 2\sqrt{2}$

A1 $x = 2.802$

A1 $x = 0.340$

3. The diagram shows the curve $(x^2 + y^2)^2 = 2(x^2 - y^2)$ and one of its maximum points M. Find the coordinates of M. [7]

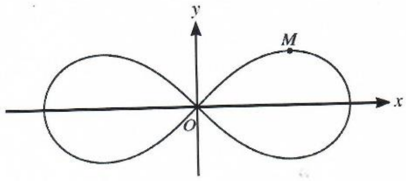

Figure 1: Curve

$2(x^2+y^2)(2x+2y\frac{dy}{dx}) = 2(2x-2y\frac{dy}{dx})$

$4x(x^2+y^2) = 4x$

$\Rightarrow x^2+y^2 = 1$ ✓

$\Rightarrow x^2-y^2 = \frac{1}{2}$

$\Rightarrow x^2 = \frac{3}{4}$ $\quad x = \frac{\sqrt{3}}{2}$

$y^2 = \frac{1}{4}$ $\quad y = \frac{1}{2}$

$M\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

4. The equation of a curve is $2x^4 + xy^3 + y^4 = 10$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{8x^3 + y^3}{3xy^2 + 4y^3}$. [4]

$8x^3 + y^3 + x \cdot 3y^2 \frac{dy}{dx} + 4y^3 \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{8x^3 + y^3}{3xy^2 + 4y^3}$

(ii) Hence show that there are two points on the curve at which the tangent is parallel to the x–axis and find the coordinates of these points. [4]

$y = -2x$ ✓

$2x^4 - 8x^4 + 16x^4 = 10$

$10x^4 = 10$

$x^4 = 1$

$x = \pm 1$

$(1, -2), (-1, 2)$

THE END